MECHANICS OF FLUIDS

COURSE CODE : MET 203

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Module 2

- Kinematics of fluid flow: Eulerian and Lagrangian approaches,
- classification of fluid flow, 1-D, 2-D and 3-D flow, steady, unsteady, uniform, non-uniform, laminar, turbulent, rotational, irrotational flows,
- stream lines, path lines, streak lines, stream tubes,
- velocity and acceleration in fluid,
- circulation and vorticity,
- stream function and potential function,
- Laplace equation,
- equipotential lines,

What Is Kinematics Of Fluid

- Kinematics of fluid flow deals with the motion of fluid particles without considering the agency producing the motion.
- This deals with the geometry of motion of fluid particles.
- This also deals with the velocity and acceleration of fluid particles in motion.

Methods of describing Fluid Motion

❖ Lagrangian Method:-

- Describes a defined mass (position, velocity, acceleration, pressure, temperature, etc.) as functions of time.
- Ex:- Track the location of a migrating bird.

FIGURE 4–1
With a small number of objects, such as billiard balls on a pool table, individual objects can be tracked.

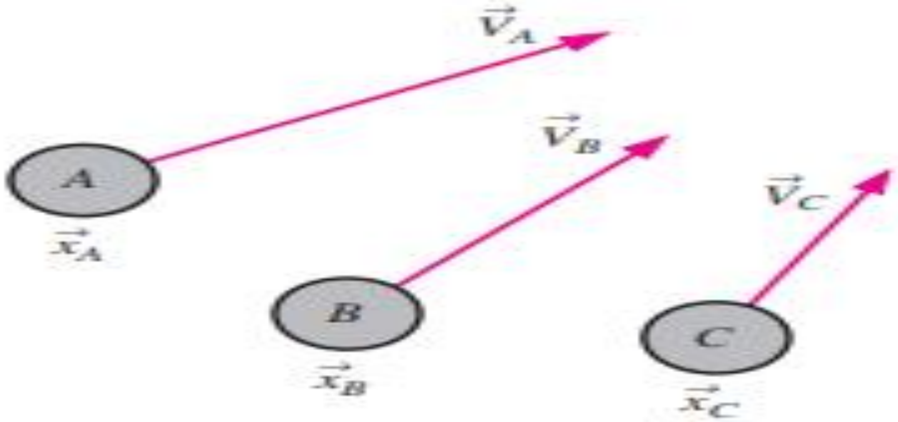


FIGURE 4–2
In the Lagrangian description, one must keep track of the position and velocity of individual particles.

Eulerian Method :-

- Describes the flow field (velocity, acceleration, pressure, temperature, etc.) as functions of position and time.

- Ex:- Count the birds passing a particular location.

THE VELOCITY FIELD

EULERIAN

Fixed Point

$p(x, y, z, t)$

LAGRANGIAN

Moving with Fluid

$p(t)$

Topics *to be discussed here*

- Types of fluid flows
- What are stream lines, path lines and streak lines

Types of flows

- Steady and unsteady flow
- Uniform and non-uniform flow
- Laminar and turbulent Flow
- Compressible and incompressible flow
- Rotational and irrotational flow
- One, two and three dimensional flow

Steady and Unsteady Flow

The flow in which fluid characteristics like velocity, pressure, density etc. at a point does not changes with time is called as steady flow.

- E.g. flow of water with constant discharge through a

pipeline is as steady flow.

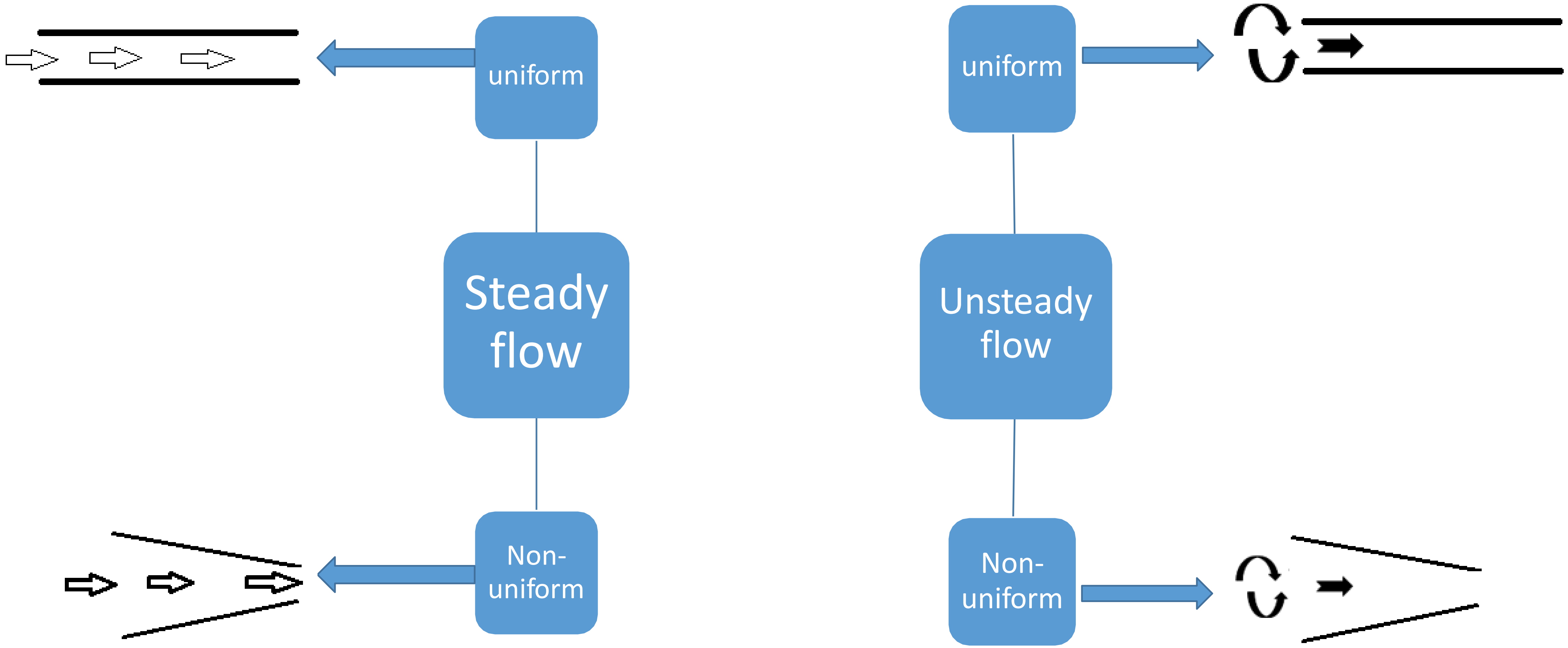
$\partial v / \partial t = 0$ $\partial P / \partial t = 0$ $\partial \rho / \partial t = 0$

The flow in which fluid characteristics like velocity, pressure, density etc. at a point changes with time is called as unsteady flow.

- E.g. flow of water with varying discharge though a pipe is

as unsteady flow.

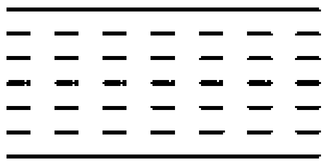
$\partial v / \partial t \neq 0$ $\partial P / \partial t \neq 0$ $\partial \rho / \partial t \neq 0$


uniform
Steady flow
Non-uniform
uniform
Unsteady flow
Non-uniform

Uniform and Non-uniform Flow

- The flow in which velocity at a given time does not change with respect to space (length of direction of flow is called as uniform flow.

E.g. flow through a long straight pipe of uniform diameter is considered as uniform flow.

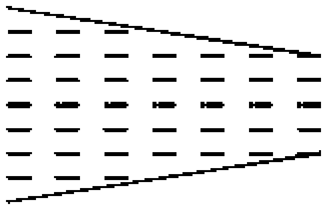


$$\partial v / \partial s = 0$$

- The flow in which velocity at a given time changes with

respect to space (length of direction of flow) is called as non-uniform flow.

E.g. flow through a long pipe with varying cross section is consider as non-uniform flow.



$$\partial v / \partial s \neq 0$$

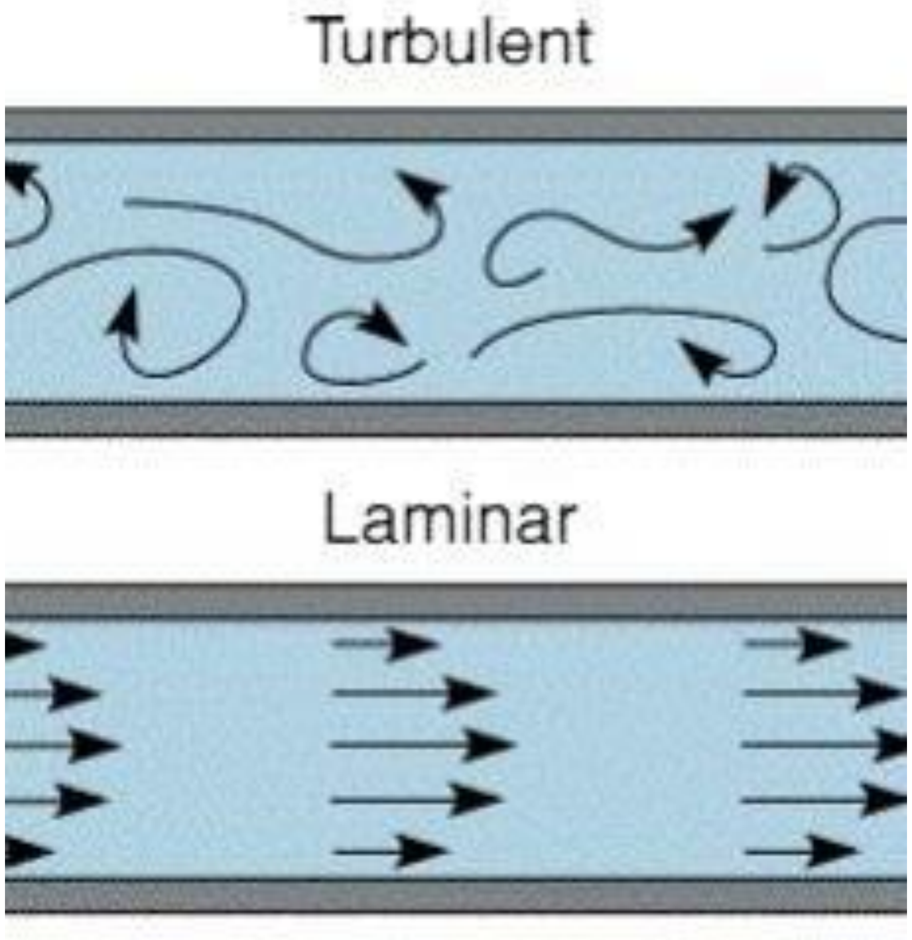
Laminar and Turbulent Flow

The flow in which the adjacent layer do not cross to each other and move along the well defined path is called as laminar flow.

E.g. flow of blood in small veins

The flow in which the adjacent layers cross each other and do not move along the well define path is called as turbulent flow.

E.g. flow through a river or canal, smoke from chimney



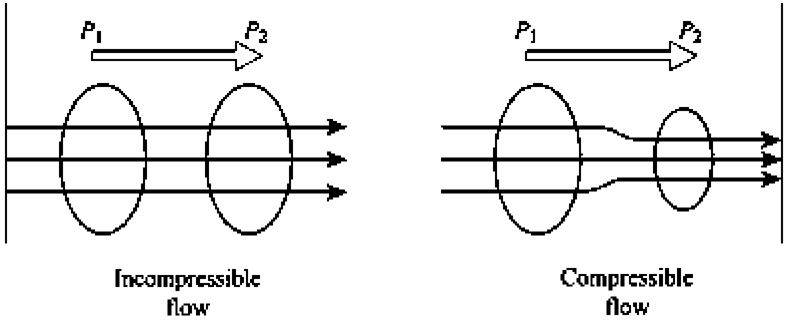
Compressible and Incompressible Flow

- The flow in which the density does not remain constant for the fluid flow is called as compressible flow.

  E.g. problems involving flight of rockets

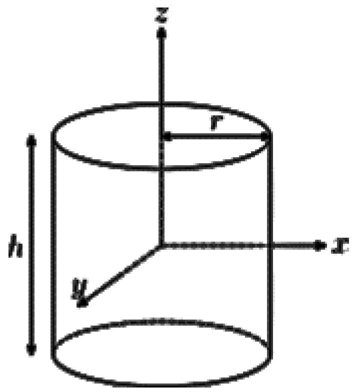
- The flow in which the density is constant for the fluid flow is called as incompressible flow.

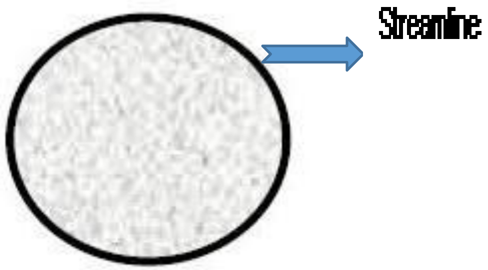
  E.g. problems involving liquids i.e. hydraulics problems



Rotational Flow

- The flow in which the fluid particle while flowing along stream lines, also rotate about their own axis is called as rotational flow.
- Consider the example of a cylinder rotating about its axis and some liquid is present inside it
- When we rotate the cylinder there is a possibility that the elements of fluid also starts rotating about its own axis when moving in a stream line such flow will be rotational





Irrotational Flow

- The flow in which the fluid particle while flowing along streamlines, do not rotate about their own axis is called as irrotational flow.
- Considering the previous example some of the fluid elements won't be rotating about its axis in a stream line
- No real flow is irrotational

One, Two and Three-dimensional Flow

1. The flow in which the velocity is the function of time and one space co-ordinate (x) is considered is called One-dimensional flow

- For example flow through the pipe is consider as a one dimensional flow

$$u = f(x),\ v = 0,\ w = 0$$

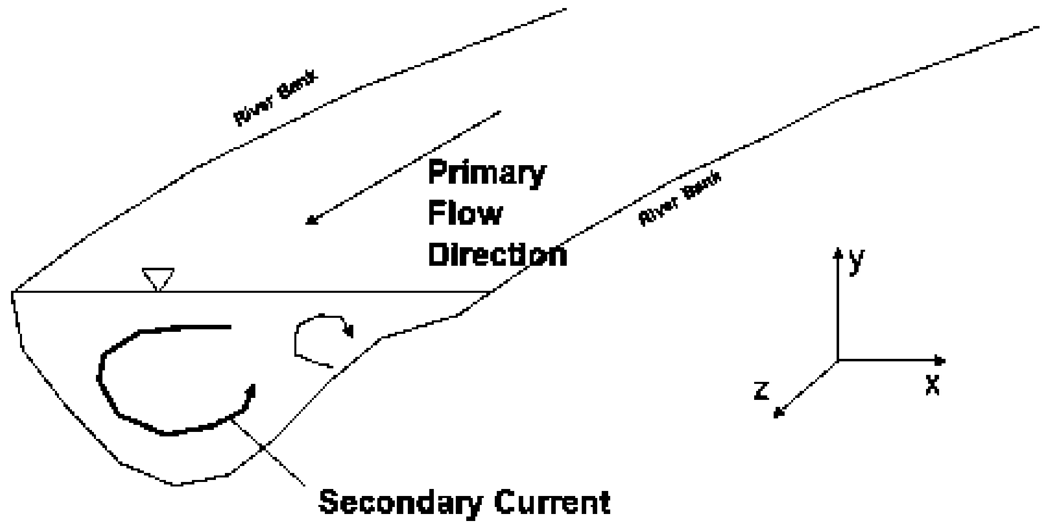
2. The flow in which the velocity is the function of time and two space co-ordinate (x,y) is considered is called two-dimensional flow.

$$u = f1(x,y),\ v = f2(x,y),\ w = 0$$

- Foe example Flow in a plane

- The flow in open channels or rivers are considered as three dimensional flow

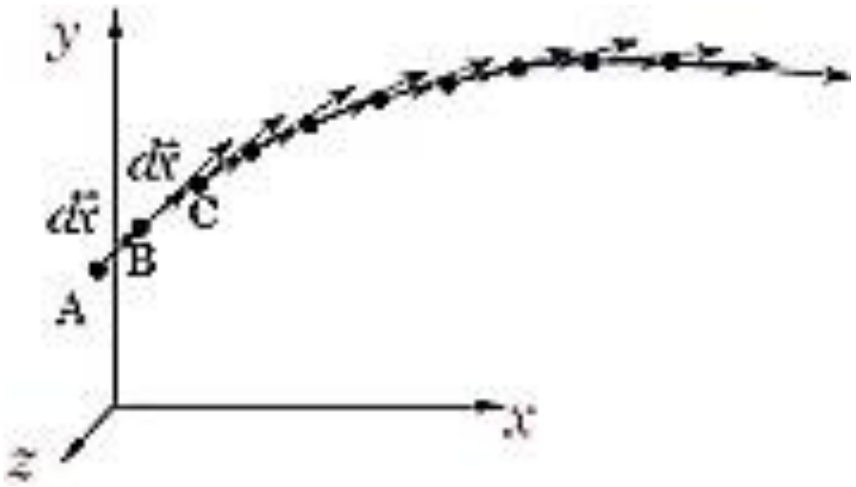
$$u = f1(x,y,z),\ v = f2(x,y,z),\ w = f3(x,y,z)$$



Truth is that when water is flowing it is almost always 3-dimensional.
But that is difficult to quantify.
So we like to simplify, if we can, to 2-dimensional or 1-dimensional
in our descriptions, analysis, and modeling.

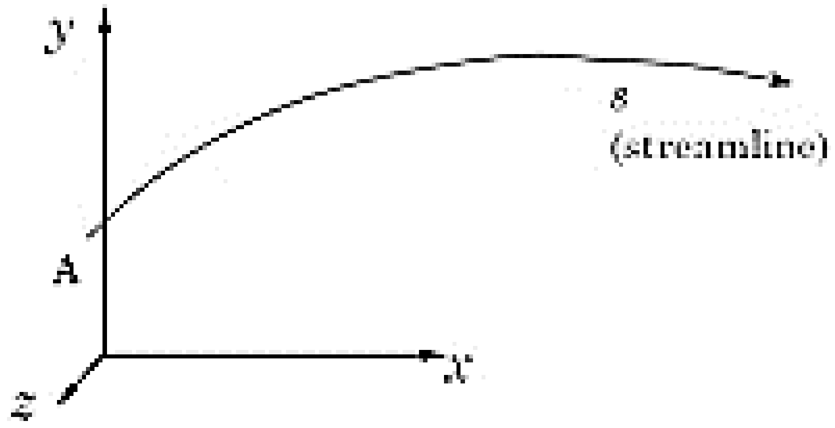
Stream lines

"It is define as an imaginary family of lines in the fluid domain that is everywhere tangent to the local velocity vector"



Assume a fluid flowing in 3D space at some instant of time let us represent the motion of fluid particles by velocity vectors now connect all this domain points with a continuous line which is tangential to all these velocity vectors

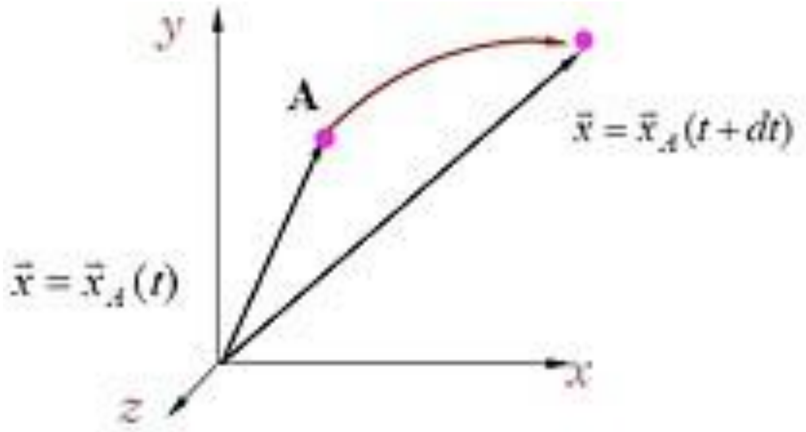
- Stream lines show the direction of fluid element at any instant of time

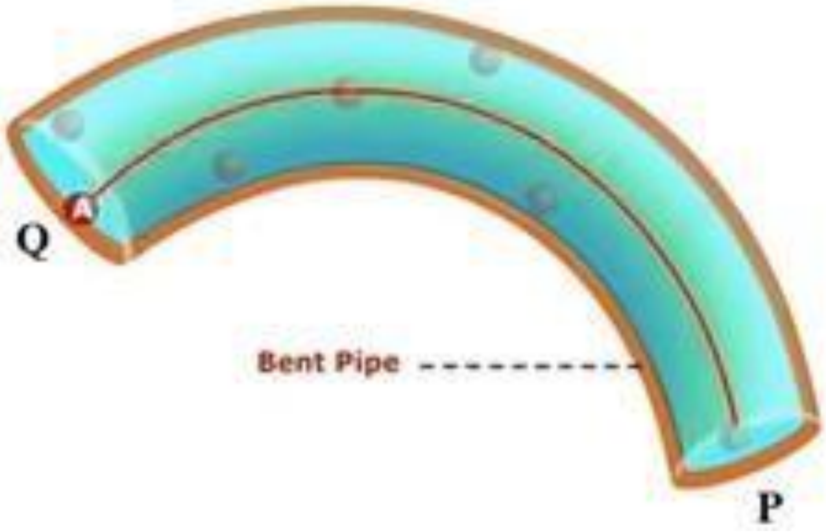


Path line

"A path line is the path or trajectory traced out by an identical fluid particle"

Consider a particle 'A' which is shown by position vector after some duration of time 'dt' it is moved to a new position, the line which is joining these two positions of point A is path line

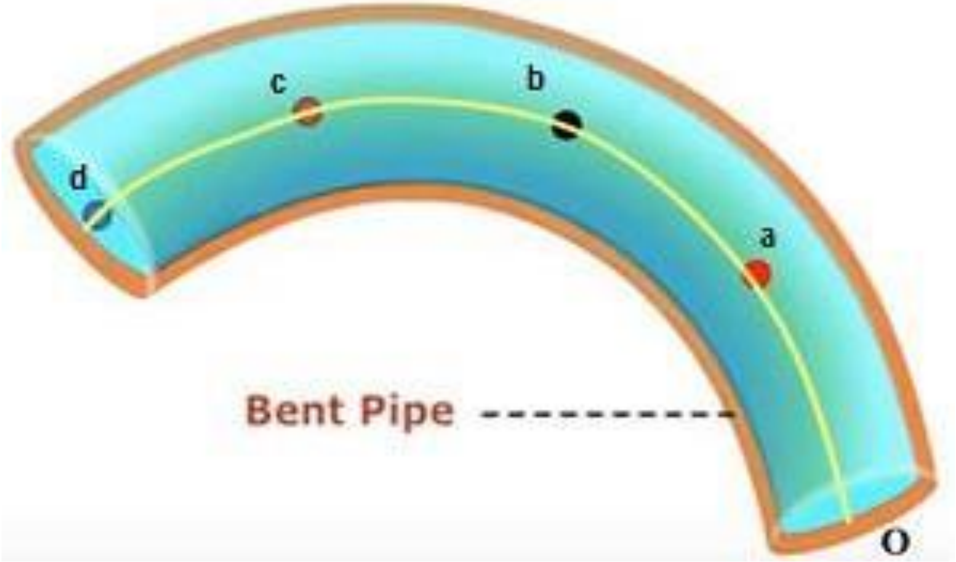




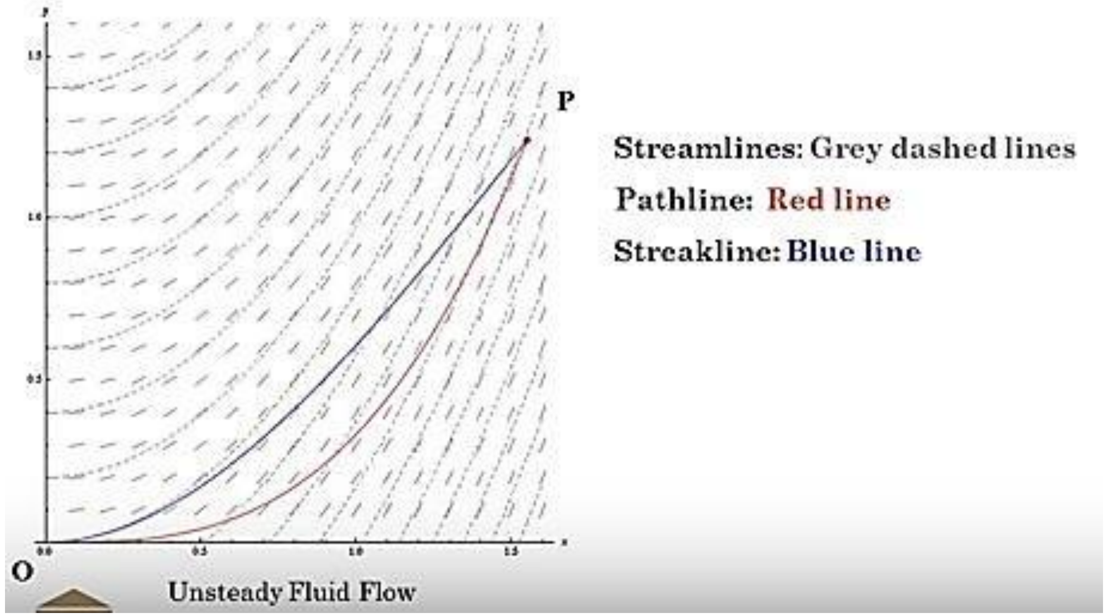
Streak lines

"A line joining the particles which earlier has passed through a fixed/common point in fluid domain is called streak line"

In figure we can see the points a,b,c,d have passed through a common point O, line joining such points is a streak line



- As we have discussed different types of lines in a fluid flow, the example which is explained here (about the bent pipe) we can see that almost all the lines were same and were showing a common trajectory the reason behind that is “in a steady flow all the lines (stream, path and streak) are same”
- For a unsteady flow they will be different and will be showing different trajectories



▶ 5.7 VELOCITY AND ACCELERATION

Let V is the resultant velocity at any point in a fluid flow. Let u, v and w are its component in x, y and z directions. The velocity components are functions of space-co-ordinates and time. Mathematically, the velocity components are given as

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

and Resultant velocity, $V = ui + vj + wk = \sqrt{u^2 + v^2 + w^2}$

Let a_x, a_y and a_z are the **total acceleration** in x, y and z directions respectively. Then by the chain rule of differentiation, we have

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial u}{\partial t}$$

But $\frac{dx}{dt} = u, \frac{dy}{dt} = v$ and $\frac{dz}{dt} = w$

$\therefore$ $$a_x = \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

Similarly, $$a_y = \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \qquad \text{...(5.6)}$$

$$a_z = \frac{dw}{dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

For steady flow, $\frac{\partial V}{\partial t} = 0$, where V is resultant velocity or $$\frac{\partial u}{\partial t} = 0, \frac{\partial v}{\partial t} = 0 \text{ and } \frac{\partial w}{\partial t} = 0$$

Hence acceleration in x, y and z directions becomes

$$\left.\begin{aligned} a_x &= \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \\ a_y &= \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \\ a_z &= \frac{dw}{dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \end{aligned}\right]$$

Acceleration vector $$\left.\begin{aligned} A &= a_x i + a_y j + a_z k \\ &= \sqrt{a_x^2 + a_y^2 + a_z^2}\,. \end{aligned}\right]$$

5.7.1 Local Acceleration and Convective Acceleration.

Local acceleration is defined as the rate of increase of velocity with respect to time at a given point in a flow field. In the equation given by (5.6), the expression $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$ or $\frac{\partial w}{\partial t}$ is known as local acceleration.

Convective acceleration is defined as the rate of change of velocity due to the change of position of fluid particles in a fluid flow. The expressions other than $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$ in equation (5.6) are known as convective acceleration.

▶ 5.4 RATE OF FLOW OR DISCHARGE (Q)

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus

(*i*) For liquids the units of Q are m^3/s or litres/s

(*ii*) For gases the units of Q is kgf/s or Newton/s

Consider a liquid flowing through a pipe in which

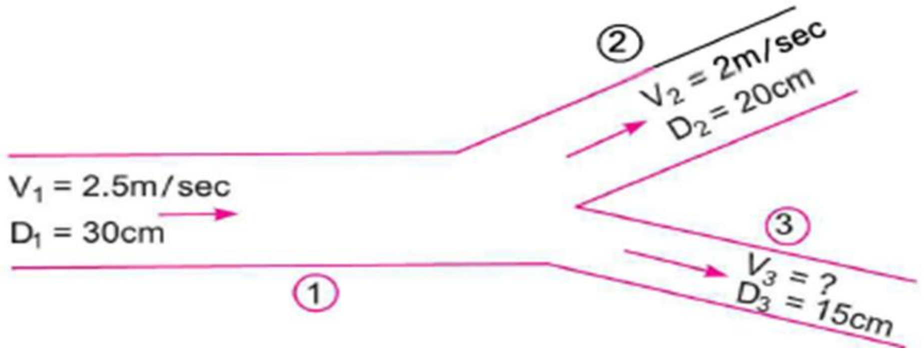
A = Cross-sectional area of pipe

V = Average velocity of fluid across the section

Then discharge $Q = A \times V$. ...(5.1)

Problem 5.2 *A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.*

Solution. Given :



$$D_1 = 30 \text{ cm} = 0.30 \text{ m}$$

$$\therefore \quad A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$$

$$V_1 = 2.5 \text{ m/s}$$

$$D_2 = 20 \text{ cm} = 0.20 \text{ m}$$

$$\therefore \quad A_2 = \frac{\pi}{4} (.2)^2 = \frac{\pi}{4} \times .4 = 0.0314 \text{ m}^2,$$

$$V_2 = 2 \text{ m/s}$$

$$D_3 = 15 \text{ cm} = 0.15 \text{ m}$$

$$\therefore \quad A_3 = \frac{\pi}{4} (.15)^2 = \frac{\pi}{4} \times 0.225 = 0.01767 \text{ m}^2$$

Find (*i*) Discharge in pipe 1 or Q_1

(*ii*) Velocity in pipe of dia. 15 cm or V_3

Let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3$$

(*i*) **The discharge $\mathbf{Q_1}$** in pipe 1 is given by

$$Q_1 = A_1V_1 = 0.07068 \times 2.5 \text{ m}^3\text{/s} = \mathbf{0.1767 \text{ m}^3\text{/s. Ans.}}$$

(*ii*) **Value of $\mathbf{V_3}$**

$$Q_2 = A_2V_2 = 0.0314 \times 2.0 = 0.0628 \text{ m}^3\text{/s}$$

Substituting the values of Q_1 and Q_2 in equation (1)

$$0.1767 = 0.0628 + Q_3$$

$$\therefore \quad Q_3 = 0.1767 - 0.0628 = 0.1139 \text{ m}^3\text{/s}$$

But $\quad Q_3 = A_3 \times V_3 = 0.01767 \times V_3 \quad \text{or} \quad 0.1139 = 0.01767 \times V_3$

$$\therefore \quad V_3 = \frac{0.1139}{0.01767} = \mathbf{6.44 \text{ m/s. Ans.}}$$

Problem 5.1 *The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.*

Solution. Given :

At section 1, $D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = 0.007854 \text{ m}^2$$

$V_1 = 5 \text{ m/s.}$

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

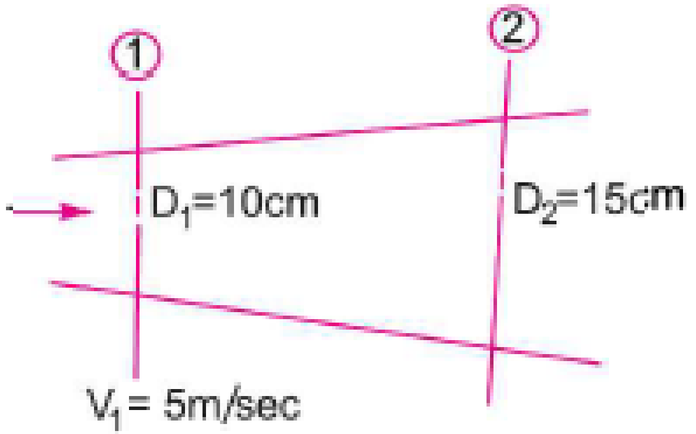


Fig. 5.2

(*i*) Discharge through pipe is given by equation (5.1)

or
$$Q = A_1 \times V_1$$
$$= 0.007854 \times 5 = \mathbf{0.03927 \text{ m}^3/\text{s}.\ Ans.}$$

Using equation (5.3), we have $A_1V_1 = A_2V_2$

(*ii*) $\therefore$
$$V_2 = \frac{A_1V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = \mathbf{2.22 \text{ m/s}.\ Ans.}$$

Continuity Equation in Fluid Mechanics

- The product of cross sectional area of the pipe and the fluid speed at any point along the pipe is constant.
- This product is equal to the volume flow per second or simply flow rate.
- **Mathematically it is represented as**

$$Av = \text{Constant}$$

- **Continuity equation derivation**

Consider a fluid flowing through a pipe of non uniform size.The particles in the fluid move along the same lines in a steady flow.

If we consider the flow for a short interval of time Δt,the fluid at the lower end of the pipe covers a distance Δx_1 with a velocity v_1 ,then:

Distance covered by the fluid = $\mathbf{\Delta x_1 = v_1 \Delta t}$

Let A_1 be the area of cross section of the lower end then volume of the fluid that flows into the pipe at the lower end $\mathbf{= V = A_1 \Delta x_1 = A_1 v_1 \Delta t}$

If ρ is the density of the fluid,then the mass of the fluid contained in the shaded region of lower end of the pipe is:

Δm_1=Density × volume

$\Delta m_1 = \rho_1 A_1 v_1 \Delta t$ ———(1)

Now the mass flux defined as the mass of the fluid per unit time passing through any cross section at lower end is:

$\Delta m_{1/\Delta t} = \rho_1 A_1 v_1$

Mass flux at lower end = **$\rho_1 A_1 v_1$** ————————(2)

If the fluid moves with velocity v_2 through the upper end of pipe having cross sectional area A_2 in time Δt,then the mass flux at the upper end is given by:

$\Delta m_{2/\Delta t} = \rho_2 A_2 v_2$

Mass flux at upper end =**$\rho_2 A_2 v_2$** ————————(3)

Since the flow is steady,so the density of the fluid between the lower and upper end of the pipe does not change with time.Thus the mass flux at the lower end must be equal to the mass flux at the upper end so:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad \text{———————}\text{-(4)}$$

In more general form we can write :

$$\rho\, A\, v = \text{constant}$$

This relation describes the law of conservation of mass in fluid dynamics.If the fluid is in compressible ,then density is constant for steady flow of in compressible fluid so

$$\rho_1 = \rho_2$$

Now equation (4) can be written as:

$$A_1 v_1 = A_2 v_2$$

In general:

$$A\, v = \text{constant}$$

5.8.2 Stream Function. It is defined as the scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. It is denoted by ψ (*Psi*) and defined only for two-dimensional flow. Mathematically, for steady flow it is defined as $\psi = f(x, y)$ such that

$$\left.\begin{aligned} \frac{\partial \psi}{\partial x} &= v \\ \text{and} \quad \frac{\partial \psi}{\partial y} &= -u \end{aligned}\right\} \qquad \text{...(5.12)}$$

The velocity components in cylindrical polar co-ordinates in terms of stream function are given as

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \text{ and } u_\theta = -\frac{\partial \psi}{\partial r} \qquad \text{...(5.12A)}$$

where u_r= radial velocity and u_θ = tangential velocity

The continuity equation for two-dimensional flow is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$

Substituting the values of u and v from equation (5.12), we get

$$\frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0 \text{ or } -\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial x \partial y} = 0.$$

Hence existence of ψ means a possible case of fluid flow. The flow may be rotational or irrotational.

The rotational component ω_z is given by $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$.

Substituting the values of u and v from equation (5.12) in the above rotational component, we get

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right] = \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

For irrotational flow, $\omega_z = 0$. Hence above equation becomes as $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial \psi}{\partial y^2} = 0$

which is Laplace equation for ψ.

The **properties** of stream function (ψ) are :

1. If stream function (ψ) exists, it is a possible case of fluid flow which may be rotational or irrotational.
2. If stream function (ψ) satisfies the Laplace equation, it is a possible case of an irrotational flow.

▶ 5.8 VELOCITY POTENTIAL FUNCTION AND STREAM FUNCTION

5.8.1 Velocity Potential Function. It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is defined by ϕ (Phi). Mathematically, the velocity, potential is defined as $\phi = f(x, y, z)$ for steady flow such that

$$\left.\begin{aligned} u &= -\frac{\partial \phi}{\partial x} \\ v &= -\frac{\partial \phi}{\partial y} \\ w &= -\frac{\partial \phi}{\partial z} \end{aligned}\right\} \qquad \text{...(5.9)}$$

where u, v and w are the components of velocity in x, y and z directions respectively.

The velocity components in cylindrical polar co-ordinates in terms of velocity potential function are given by

$$\left.\begin{aligned} u_r &= \frac{\partial \phi}{\partial r} \\ u_\theta &= \frac{1}{r}\frac{\partial \phi}{\partial \theta} \end{aligned}\right\} \qquad \text{...(5.9}A\text{)}$$

where u_r = velocity component in radial direction (*i.e.*, in r direction)

and u_θ = velocity component in tangential direction (*i.e.*, in θ direction)

The continuity equation for an incompressible steady flow is $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0.$

Substituting the values of u, v and w from equation (5.9), we get

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right)=0$$

or

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}=0. \qquad \text{...(5.10)}$$

Equation (5.10) is a Laplace equation.

For two-dimension case, equation (5.10) reduces to $\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=0.$ ...(5.11)

1. If velocity potential (ϕ) exists, the flow should be irrotational.
2. If velocity potential (ϕ) satisfies the Laplace equation, it represents the possible steady incompressible irrotational flow.

Problem 5.6 *The velocity vector in a fluid flow is given*

$$V = 4x^3 i - 10x^2 yj + 2tk.$$

Find the velocity and acceleration of a fluid particle at (2, 1, 3) at time t = 1.

Solution. The velocity components u, v and w are $u = 4x^3$, $v = -10x^2 y$, $w = 2t$

For the point (2, 1, 3), we have $x = 2$, $y = 1$ and $z = 3$ at time $t = 1$.

Hence velocity components at (2, 1, 3) are

$$u = 4 \times (2)^3 = 32 \text{ units}$$

$$v = -10(2)^2(1) = -40 \text{ units}$$

$$w = 2 \times 1 = 2 \text{ units}$$

∴ Velocity vector V at (2, 1, 3) $= 32i - 40j + 2k$

or Resultant velocity $= \sqrt{u^2 + v^2 + w^2}$

$$= \sqrt{32^2 + (-40)^2 + 2^2} = \sqrt{1024 + 1600 + 4} = \mathbf{51.26 \text{ units.}}$$ /

Acceleration is given by equation (5.6)

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Now from velocity components, we have

$$\frac{\partial u}{\partial x} = 12x^2, \ \frac{\partial u}{\partial y} = 0, \ \frac{\partial u}{\partial z} = 0 \text{ and } \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial v}{\partial x} = -20xy, \ \frac{\partial v}{\partial y} = -10x^2, \ \frac{\partial v}{\partial z} = 0 \ \frac{\partial v}{\partial t} = 0$$

$$\frac{\partial w}{\partial x} = 0, \ \frac{\partial w}{\partial y} = 0, \frac{\partial w}{\partial z} = 0 \text{ and } \frac{\partial w}{\partial t} = 2.1$$

Substituting the values, the acceleration components at (2, 1, 3) at time $t = 1$ are

$$a_x = 4x^3(12x^2) + (-10x^2y)(0) + 2t \times (0) + 0$$

$$= 48x^5 = 48 \times (2)^5 = 48 \times 32 = 1536 \text{ units}$$

$$a_y = 4x^3(-20xy) + (-10x^2y)(-10x^2) + 2t(0) + 0$$

$$= -80x^4y + 100x^4y$$

$$= -80(2)^4(1) + 100(2)^4 \times 1 = -1280 + 1600 = 320 \text{ units.}$$

$$a_z = 4x^3(0) + (-10x^2y)(0) + (2t)(0) + 2.1 = 2.0 \text{ units}$$

∴ Acceleration is $A = a_x i + a_y j + a_z k = \mathbf{1536i + 320j + 2k.}$ **Ans.**

or Resultant $A = \sqrt{(1536)^2 + (320)^2 + (2)^2}$ units

$$= \sqrt{2359296 + 102400 + 4} = \mathbf{1568.9 \text{ units.}}$$ **Ans.**

Problem 5.7 *The following cases represent the two velocity components, determine the third component of velocity such that they satisfy the continuity equation :*

(i) $u = x^2 + y^2 + z^2$; $v = xy^2 - yz^2 + xy$

(ii) $v = 2y^2$, $w = 2xyz$.

Solution. The continuity equation for incompressible fluid is given by equation (5.4) as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Case I. $u = x^2 + y^2 + z^2 \quad \therefore \quad \frac{\partial u}{\partial x} = 2x$

$$v = xy^2 - yz^2 + xy \quad \therefore \quad \frac{\partial v}{\partial y} = 2xy - z^2 + x$$

Substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation.

$$2x + 2xy - z^2 + x + \frac{\partial w}{\partial z} = 0$$

or $$\frac{\partial w}{\partial z} = -3x - 2xy + z^2 \text{ or } \partial w = (-3x - 2xy + z^2)\,\partial z$$

Integration of both sides gives $\int dw = \int(-3x - 2xy + z^2)\,dz$

or $$w = \left(-3xz - 2xyz + \frac{z^3}{3}\right) + \text{Constant of integration,}$$

where constant of integration cannot be a function of z. But it can be a function of x and y that is $f(x, y)$.

$\therefore$ $$w = \left(\mathbf{-3xz - 2xyz + \frac{z^3}{3}}\right) + \mathbf{f(x, y)}. \textbf{ Ans.}$$

Case II. $v = 2y^2 \quad \therefore \quad \frac{\partial v}{\partial y} = 4y$

$$w = 2xyz \quad \therefore \quad \frac{\partial w}{\partial z} = 2xy$$

Substituting the values of $\frac{\partial v}{\partial y}$ and $\frac{\partial w}{\partial z}$ in continuity equation, we get

$$\frac{\partial u}{\partial x} + 4y + 2xy = 0$$

or $$\frac{\partial u}{\partial x} = -4y - 2xy \text{ or } du = (-4y - 2xy)\,dx$$

Integrating, we get $$u = -4xy - 2y\frac{x^2}{2} + f(y, z) = \mathbf{-4xy - x^2y + f(y, z)}. \textbf{ Ans.}$$

Problem 5.8 *A fluid flow field is given by*

$$V = x^2yi + y^2zj - (2xyz + yz^2)k$$

Prove that it is a case of possible steady incompressible fluid flow. Calculate the velocity and acceleration at the point (2, 1, 3).

Solution. For the given fluid flow field $u = x^2y \qquad \therefore \quad \frac{\partial u}{\partial x} = 2xy$

$$v = y^2z \qquad \therefore \quad \frac{\partial v}{\partial y} = 2yz$$

$$w = -2xyz - yz^2 \qquad \therefore \quad \frac{\partial w}{\partial z} = -2xy - 2yz.$$

For a case of possible steady incompressible fluid flow, the continuity equation (5.4) should be satisfied.

i.e.,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

Substituting the values of $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$ and $\frac{\partial w}{\partial z}$, we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 2xy + 2yz - 2xy - 2yz = 0$$

Hence the velocity field $V = x^2yi + y^2zj - (2xyz + yz^2)\,k$ is a possible case of fluid flow. **Ans.**

Velocity at (2, 1, 3)

Substituting the values $x = 2$, $y = 1$ and $z = 3$ in velocity field, we get

$$V = x^2yi + y^2zj - (2xyz + yz^2)\,k$$
$$= 2^2 \times 1i + 1^2 \times 3j - (2 \times 2 \times 1 \times 3 + 1 \times 3^2)\,k$$
$$= \mathbf{4i + 3j - 21k}. \text{ **Ans.**}$$

and Resultant velocity $= \sqrt{4^2 + 3^2 + (-21)^2} = \sqrt{16 + 9 + 441} = \sqrt{466} =$ **21.587 units. Ans.**

Acceleration at (2, 1, 3)

The acceleration components a_x, a_y and a_z for steady flow are

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$u = x^2y, \frac{\partial u}{\partial x} = 2xy, \frac{\partial u}{\partial y} = x^2 \text{ and } \frac{\partial u}{\partial z} = 0$$

$$v = y^2z, \frac{\partial v}{\partial x} = 0, \frac{\partial v}{\partial y} = 2yz, \frac{\partial v}{\partial z} = y^2$$

$$w = -2xyz - yz^2, \frac{\partial w}{\partial x} = -2yz, \frac{\partial w}{\partial y} = -2xz - z^2, \frac{\partial w}{\partial z} = -2xy - 2yz.$$

Substituting these values in acceleration components, we get acceleration at (2, 1, 3)

$$a_x = x^2y\,(2xy) + y^2z\,(x)^2 - (2xyz + yz^2)\,(0)$$
$$= 2x^3y^2 + x^2y^2z$$
$$= 2\,(2)^3 1^2 + 2^2 \times 1^2 \times 3 = 2 \times 8 + 12$$
$$= 16 + 12 = 28 \text{ units}$$

$$a_y = x^2y\,(0) + y^2z\,(2yz) - (2xyz + yz^2)\,(y^2)$$
$$= 2y^3z^2 - 2xy^3z - y^3z^2$$
$$= 2 \times 1^3 \times 3^2 - 2 \times 2 \times 1^3 \times 3 - 1^3 \times 3^2 = 18 - 12 - 9 = -3 \text{ units}$$

$$a_z = x^2y\,(-2yz) + y^2z\,(-2xz - z^2) - (2xyz + yz^2)\,(-2xy - 2yz)$$
$$= -2x^2y^2z - 2xy^2z^2 - y^2z^3 + [4x^2y^2z + 2xy^2z^2 + 4xy^2z^2 + 2y^2z^3]$$
$$= -2 \times 2^2 \times 1^2 \times 3 - 2 \times 2 \times 1^2 \times 3^2 - 1^2 \times 3^3$$
$$+ [4 \times 2^2 \times 1^2 \times 3 + 2 \times 2 \times 1^2 \times 3^2 + 4 \times 2 \times 1^2 \times 3^2 + 2 \times 1^2 \times 3^3]$$
$$= -24 - 36 - 27 + [48 + 36 + 72 + 54]$$
$$= -24 - 36 - 27 + 48 + 36 + 72 + 54 = 123$$

$\therefore$ Acceleration $= a_xi + a_yj + a_zk =$ **28i – 3j + 123k. Ans.**

or Resultant acceleration $= \sqrt{28^2 + (-3)^2 + 123^2} = \sqrt{784 + 9 + 15129}$

$= \sqrt{15922} =$ **126.18 units. Ans.**

Problem 5.9 *Find the convective acceleration at the middle of a pipe which converges uniformly from 0.4 m diameter to 0.2 m diameter over 2 m length. The rate of flow is 20 lit/s. If the rate of flow changes uniformly from 20 l/s to 40 l/s in 30 seconds, find the total acceleration at the middle of the pipe at 15th second.*

Solution. Given :

Diameter at section 1, $D_1 = 0.4$ m ; $D_2 = 0.2$ m, $L = 2$ m, $Q = 20$ *l*/s $= 0.02$ m^3/s as one litre $= 0.001\text{m}^3 = 1000$ cm^3

Find (*i*) Convective acceleration at middle *i.e.*, at *A* when $Q = 20$ *l*/s.

(*ii*) Total acceleration at *A* when *Q* changes from 20 *l*/s to 40 *l*/s in 30 seconds.

Case I. In this case, the rate of flow is constant and equal to 0.02 m^3/s. The velocity of flow is in *x*-direction only. Hence this is one-dimensional flow and velocity components in *y* and *z* directions are zero or $v = 0$, $z = 0$.

$$\therefore \quad \text{Convective acceleration} = u\frac{\partial u}{\partial y} \text{ only} \qquad \ldots(i)$$

Let us find the value of u and $\frac{\partial u}{\partial x}$ at a distance x from inlet

The diameter (D_x) at a distance x from inlet or at section *X-X* is given by,

$$D_x = 0.4 - \frac{0.4 - 0.2}{2} \times x$$

$$= (0.4 - 0.1\,x) \text{ m}$$

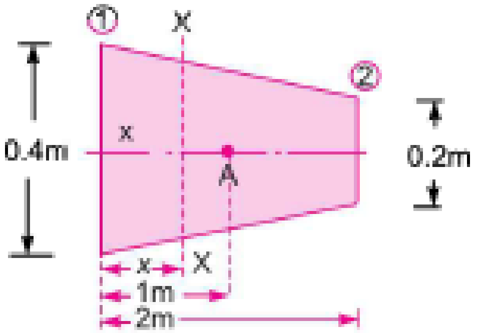


Fig. 5.8

The area of cross-section (A_x) at section *X-X* is given by,

$$A_x = \frac{\pi}{4} D_x^2 = \frac{\pi}{4}(0.4 - 0.1\,x)^2$$

Velocity (u) at the section *X-X* in terms of Q (*i.e.*, in terms of rate of flow)

$$u = \frac{Q}{\text{Area}} = \frac{Q}{A_x} = \frac{Q}{\frac{\pi}{4}D_x^2} = \frac{4Q}{\pi\,(0.4 - 0.1x)^2}$$

$$= \frac{1.273Q}{(0.4 - 0.1x)^2} = 1.273\,Q\,(0.4 - 0.1\,x)^{-2} \text{ m/s} \qquad \ldots(ii)$$

To find $\frac{\partial u}{\partial x}$, we must differentiate equation (*ii*) with respect to *x*.

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}[1.273\, Q\, (0.4 - 0.1\, x)^{-2}]$$

$$= 1.273\, Q\, (-2)\, (0.4 - 0.1\, x)^{-1} \times (-0.1) \qquad \text{[Here Q is constant]}$$

$$= 0.2546\, Q\, (0.4 - 0.1\, x)^{-1} \qquad ...(iii)$$

Substituting the value of *u* and $\frac{\partial u}{\partial x}$ in equation (*i*), we get

$$\text{Convective acceleration} = [1.273\, Q\, (0.4 - 0.1\, x)^{-2}] \times [0.2546\, Q\, (0.4 - 0.1\, x)^{-1}]$$

$$= 1.273 \times 0.2546 \times Q^2 \times (0.4 - 0.1\, x)^{-3}$$

$$= 1.273 \times 0.2546 \times (0.02)^2 \times (0.4 - 0.1\, x)^{-3} \quad [\because Q = 0.02 \text{ m}^3/\text{s}]$$

∴ Convective acceleration at the middle (where $x = 1$ m)

$$= 1.273 \times 0.2546 \times (0.02)^2 \times (0.4 - 0.1 \times 1)^{-3} \text{ m/s}^2$$

$$= 1.273 \times 0.2546 \times (0.02)^2 \times (0.3)^{-3} \text{ m/s}^2$$

$$= \mathbf{0.0048\ m/s^2.\ Ans.}$$

Case II. When Q changes from 0.02 m^3/s to 0.04 m^3/s in 30 seconds, find the total acceleration at $x = 1$ m and $t = 15$ seconds.

Total acceleration = Convective acceleration + Local acceleration at $t = 15$ seconds.

The rate of flow at $t = 15$ seconds is given by

$$Q = Q_1 + \frac{Q_2 - Q_1}{30} \times 15 \text{ where } Q_2 = 0.04 \text{ m}^3/\text{s and } Q_1 = 0.02 \text{ m}^3/\text{s}$$

$$= 0.02 + \frac{(0.04 - 0.02)}{30} \times 15 = 0.03 \text{ m}^3/\text{s}$$

The velocity (*u*) and gradient $\left(\frac{\partial u}{\partial x}\right)$ in terms of Q are given by equations (*ii*) and (*iii*) respectively

$$\therefore \text{Convective acceleration} = u.\frac{\partial u}{\partial x}$$

$$= [1.273\, Q\, (0.4 - 0.1\, x)^{-2}] \times [0.2546\, Q\, (0.4 - 0.1\, x)^{-1}]$$

$$= 1.273 \times 0.2546\, Q^2 \times (0.4 - 0.1 \times 1)^{-3}$$

∴ Convective acceleration (when $Q = 0.03$ m^3/s and $x = 1$ m)

$$= 1.273 \times 0.2546 \times (0.03)^2 \times (0.4 - 0.1 \times 1)^{-3}$$

$$= 1.273 \times 0.2546 \times (0.03)^2 \times (0.3)^{-3} \text{ m/s}^2$$

$$= 0.0108 \text{ m/s}^2 \qquad ...(iv)$$

$$\text{Local acceleration} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}[1.273\, Q\, (0.4 - 0.1\, x)^{-2}]$$

[∵ u from equation (*ii*) is $u = 1.273\, Q\, (0.4 - 0.1\, x)^{-2}$]

$$= 1.273 \times (0.4 - 0.1\, x)^{-2} \times \frac{\partial Q}{\partial t}$$

[∵ Local acceleration is at a point where x is constant but Q is changing]

Local acceleration (at $x = 1$ m)

$$= 1.273 \times (0.4 - 0.1 \times 1)^{-2} \times \frac{\partial Q}{\partial t}$$

$$= 1.273 \times (0.3)^{-2} \times \frac{0.02}{30} \qquad \left[\because \frac{\partial Q}{\partial t} = \frac{Q_2 - Q_1}{t} = \frac{0.04 - 0.02}{30} = \frac{0.02}{30}\right]$$

$$= 0.00943 \text{ m/s}^2 \qquad \ldots(v)$$

Hence adding equations (*iv*) and (*v*), we get total acceleration.

∴ Total acceleration = Convective acceleration + Local acceleration

$$= 0.0108 + 0.00943 = \mathbf{0.02023\ m/s^2}. \ \textbf{Ans.}$$

Problem 1.6 *Determine the intensity of shear of an oil having viscosity = 1 poise. The oil is used for lubricating the clearance between a shaft of diameter 10 cm and its journal bearing. The clearance is 1.5 mm and the shaft rotates at 150 r.p.m.*

Solution. Given : $\mu = 1 \text{ poise} = \frac{1}{10} \frac{\text{N s}}{\text{m}^2}$

Dia. of shaft, $D = 10 \text{ cm} = 0.1 \text{ m}$

Distance between shaft and journal bearing,

$$dy = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

Speed of shaft, $N = 150 \text{ r.p.m.}$

Tangential speed of shaft is given by

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.1 \times 150}{60} = 0.785 \text{ m/s}$$

Using equation (1.2), $\tau = \mu \frac{du}{dy}$,

where du = change of velocity between shaft and bearing = $u - 0 = u$

$$= \frac{1}{10} \times \frac{0.785}{1.5 \times 10^{-3}} = \mathbf{52.33 \text{ N/m}^2}\text{. **Ans.**}$$

Problem 1.4 *A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m² to maintain this speed. Determine the fluid viscosity between the plates.*

Solution. Given :

Distance between plates, $dy = .025$ mm $= .025 \times 10^{-3}$ m

Velocity of upper plate, $u = 60$ cm/s $= 0.6$ m/s

Force on upper plate, $F = 2.0 \dfrac{\text{N}}{\text{m}^2}$.

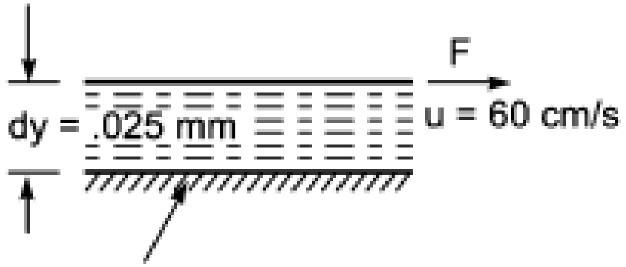


Fig. 1.3

This is the value of shear stress *i.e.*, τ

Let the fluid viscosity between the plates is μ.

Using the equation (1.2), we have $\tau = \mu \dfrac{du}{dy}$.

where du = Change of velocity $= u - 0 = u = 0.60$ m/s

dy = Change of distance $= .025 \times 10^{-3}$ m

τ = Force per unit area $= 2.0 \dfrac{\text{N}}{\text{m}^2}$

$$\therefore \quad 2.0 = \mu \frac{0.60}{.025 \times 10^{-3}} \quad \therefore \quad \mu = \frac{2.0 \times .025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \frac{\text{Ns}}{\text{m}^2}$$

$= 8.33 \times 10^{-5} \times 10$ poise $=$ **8.33×10^{-4} poise. Ans.**

Problem 1.7 *Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m × 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.*

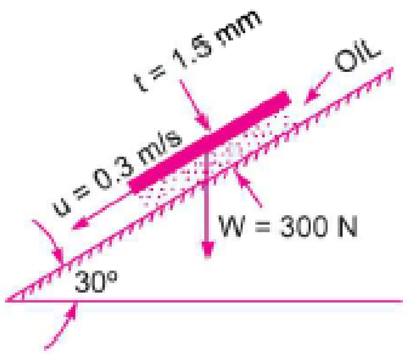


Fig. 1.4

Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300$ N

Velocity of plate, $u = 0.3$ m/s

---

* Power = $F \times u$ N m/s = $F \times u$ W (∵ Nm/s = Watt)

Thickness of oil film, $t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane = $W \cos 60° = 300 \cos 60° = 150$ N

Thus the shear force, F, on the bottom surface of the plate = 150 N

and shear stress, $$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64} \text{ N/m}^2$$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity = $u - 0 = u = 0.3$ m/s

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = \textbf{11.7 poise. Ans.}$$

Problem 1.9 *The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed. Determine :*

(i) the dynamic viscosity of the oil in poise, and

(ii) the kinematic viscosity of the oil in stokes if the specific gravity of the oil is 0.95.

Solution. Given :

Each side of a square plate $= 60 \text{ cm} = 0.60 \text{ m}$

∴ Area, $A = 0.6 \times 0.6 = 0.36 \text{ m}^2$

Thickness of oil film, $dy = 12.5 \text{ mm} = 12.5 \times 10^{-3} \text{ m}$

Velocity of upper plate, $u = 2.5 \text{ m/sec}$

∴ Change of velocity between plates, $du = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

∴ Shear stress, $\tau = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{A} = \dfrac{98.1 \text{ N}}{0.36 \text{ m}^2}$

(*i*) Let μ = Dynamic viscosity of oil

Using equation (1.2), $\tau = \mu \dfrac{du}{dy}$ or $\dfrac{98.1}{0.36} = \mu \times \dfrac{2.5}{12.5 \times 10^{-3}}$

$$\therefore \quad \mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635 \frac{\text{Ns}}{\text{m}^2} \qquad \left(\because \frac{1 \text{ Ns}}{\text{m}^2} = 10 \text{ poise}\right)$$

$= 1.3635 \times 10 =$ **13.635 poise. Ans.**

(*ii*) Sp. gr. of oil, S = 0.95

Let ν = kinematic viscosity of oil

Using equation (1.1*A*),

Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950 \text{ kg/m}^3$

Using the relation, $\nu = \dfrac{\mu}{\rho}$, we get $\nu = \dfrac{1.3635 \left(\dfrac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435 \text{ m}^2\text{/sec} = .001435 \times 10^4 \text{ cm}^2\text{/s}$

= **14.35 stokes. Ans.** $(\because \text{cm}^2\text{/s} = \text{stoke})$

Problem 1.14 *The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 r.p.m. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.*

Solution. Given :

Viscosity $\mu = 6$ poise

$$= \frac{6}{10} \frac{\text{N s}}{\text{m}^2} = 0.6 \frac{\text{N s}}{\text{m}^2}$$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ r.p.m

Sleeve length, $L = 90 \text{ mm} = 90 \times 10^{-3}$ m

Thickness of oil film, $t = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

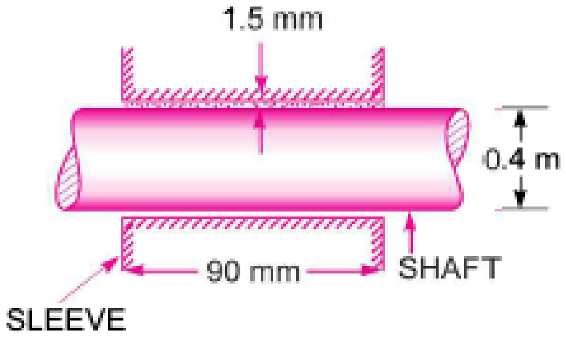


Fig. 1.5

Tangential velocity of shaft, $u = \dfrac{\pi D N}{60} = \dfrac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

Using the relation $\tau = \mu \dfrac{du}{dy}$

where du = Change of velocity $= u - 0 = u = 3.98$ m/s

dy = Change of distance $= t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

$\therefore$ Shear force on the shaft, F = Shear stress $\times$ Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft, $T = \text{Force} \times \dfrac{D}{2} = 180.05 \times \dfrac{0.4}{2} = 36.01$ Nm

$\therefore$ *Power lost $= \dfrac{2\pi NT}{60} = \dfrac{2\pi \times 190 \times 36.01}{60} =$ **716.48 W. Ans.**

Problem 1.15 *If the velocity profile of a fluid over a plate is parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stresses at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.*

Solution. Given :

Distance of vertex from plate = 20 cm

Velocity at vertex, $u = 120$ cm/sec

Viscosity, $\mu = 8.5 \text{ poise} = \dfrac{8.5}{10} \dfrac{\text{N s}}{\text{m}^2} = 0.85.$

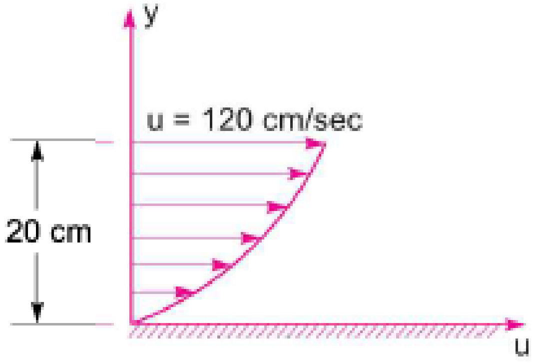


Fig. 1.6

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* Power in S.I. unit = T * ω = $T \times \dfrac{2\pi N}{60}$ Watt = $\dfrac{2\pi NT}{60}$ Watt

The velocity profile is given parabolic and equation of velocity profile is

$$u = ay^2 + by + c \quad \text{...(i)}$$

where a, b and c are constants. Their values are determined from boundary conditions as :

(a) at $y = 0$, $u = 0$

(b) at $y = 20$ cm, $u = 120$ cm/sec

(c) at $y = 20$ cm, $\frac{du}{dy} = 0$.

Substituting boundary condition (a) in equation (i), we get

$$c = 0.$$

Boundary condition (b) on substitution in (i) gives

$$120 = a(20)^2 + b(20) = 400a + 20b \quad \text{...(ii)}$$

Boundary condition (c) on substitution in equation (i) gives

$$\frac{du}{dy} = 2ay + b \quad \text{...(iii)}$$

or
$$0 = 2 \times a \times 20 + b = 40a + b$$

Solving equations (ii) and (iii) for a and b

From equation (iii), $b = -40a$

Substituting this value in equation (ii), we get

$$120 = 400a + 20 \times (-40a) = 400a - 800a = -400a$$

$$\therefore \quad a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$$\therefore \quad b = -40 \times (-0.3) = 12.0$$

Substituting the values of a, b and c in equation (i),

$$u = -0.3y^2 + 12y.$$

Velocity Gradient

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

at $y = 0$, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 =$ **12/s. Ans.**

at $y = 10$ cm, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 =$ **6/s. Ans.**

at $y = 20$ cm, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 =$ **0. Ans.**

Shear Stresses

Shear stress is given by, $\tau = \mu \frac{du}{dy}$

(*i*) Shear stress at $y = 0$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.2 \text{ N/m}^2$.

(*ii*) Shear stress at $y = 10$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6.0 = 5.1 \text{ N/m}^2$.

(*iii*) Shear stress at $y = 20$, $\tau = \mu \left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0 =$ **0. Ans.**

Problem 5.19 *The velocity components in a two-dimensional flow are*

$$u = y^3/3 + 2x - x^2y \text{ and } v = xy^2 - 2y - x^3/3.$$

Show that these components represent a possible case of an irrotational flow.

Solution. Given : $u = y^3/3 + 2x - x^2y$

$$\therefore \quad \frac{\partial u}{\partial x} = 2 - 2xy$$

$$\frac{\partial u}{\partial y} = \frac{3y^2}{3} - x^2 = y^2 - x^2$$

Also $v = xy^2 - 2y - x^3/3$

$$\therefore \quad \frac{\partial v}{\partial y} = 2xy - 2$$

$$\frac{\partial v}{\partial x} = y^2 - \frac{3x^2}{3} = y^2 - x^2.$$

(*i*) For a two-dimensional flow, continuity equation is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Substituting the value of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$, we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2 - 2xy + 2xy - 2 = 0$$

$\therefore$ It is a possible case of fluid flow.

(*ii*) Rotation, ω_z is given by $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\,[(y^2 - x^2) - (y^2 - x^2)] = 0$

$\therefore$ Rotation is zero, which means it is case of irrotational flow. **Ans.**

Problem 5.16 *The velocity components in a two-dimensional flow field for an incompressible fluid are as follows :*

$$u = \frac{y^3}{3} + 2x - x^2y \text{ and } v = xy^2 - 2y - x^3/3$$

obtain an expression for the stream function ψ.

Solution. Given : $u = y^3/3 + 2x - x^2y$

$v = xy^2 - 2y - x^3/3.$

The velocity components in terms of stream function are

$$\frac{\partial \psi}{\partial x} = v = xy^2 - 2y - x^3/3 \quad \text{...(i)}$$

$$\frac{\partial \psi}{\partial y} = -u = -y^3/3 - 2x + x^2y \quad \text{...(ii)}$$

Integrating (*i*) w.r.t. x, we get $\psi = \int (xy^2 - 2y - x^3/3)\, dx$

or
$$\psi = \frac{x^2y^2}{2} - 2xy - \frac{x^4}{4 \times 3} + k, \quad \text{...(iii)}$$

where k is a constant of integration which is independent of x but can be a function of y.

Differentiating equation (*iii*) w.r.t. y, we get

$$\frac{\partial \psi}{\partial y} = \frac{2x^2y}{2} - 2x + \frac{\partial k}{\partial y} = x^2y - 2x + \frac{\partial k}{\partial y}$$

But from (*ii*),
$$\frac{\partial \psi}{\partial y} = -y^3/3 - 2x + x^2y$$

Comparing the value of $\frac{\partial \psi}{\partial y}$, we get $x^2y - 2x + \frac{\partial k}{\partial y} = -y^3/3 - 2x + x^2y$

$$\therefore \quad \frac{\partial k}{\partial y} = -y^3/3$$

Integrating, we get
$$k = \int (-y^3/3)\, dy = \frac{-y^4}{4 \times 3} = \frac{-y^4}{12}$$

Substituting this value in (*iii*), we get

$$\psi = \frac{x^2y^2}{2} - 2xy - \frac{x^4}{12} - \frac{y^4}{12}. \textbf{ Ans.}$$

Problem 2.2 *A hydraulic press has a ram of 20 cm diameter and a plunger of 3 cm diameter. It is used for lifting a weight of 30 kN. Find the force required at the plunger.*

Solution. Given :

Dia. of ram, $D = 20 \text{ cm} = 0.2 \text{ m}$

∴ Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$

Dia. of plunger $d = 3 \text{ cm} = 0.03 \text{ m}$

∴ Area of plunger, $a = \frac{\pi}{4} (.03)^2 = 7.068 \times 10^{-4} \text{ m}^2$

Weight lifted, $W = 30 \text{ kN} = 30 \times 1000 \text{ N} = 30000 \text{ N}.$

See Fig. 2.3.

Pressure intensity developed due to plunger $= \frac{\text{Force}}{\text{Area}} = \frac{F}{a}.$

By Pascal's Law, this pressure is transmitted equally in all directions

Hence pressure transmitted at the ram $= \frac{F}{a}$

∴ Force acting on ram = Pressure intensity × Area of ram

$$= \frac{F}{a} \times A = \frac{F \times .0314}{7.068 \times 10^{-4}} \text{ N}$$

But force acting on ram = Weight lifted = 30000 N

$$\therefore \quad 30000 = \frac{F \times .0314}{7.068 \times 10^{-4}}$$

$$\therefore \quad F = \frac{30000 \times 7.068 \times 10^{-4}}{.0314} = \textbf{675.2 N. Ans.}$$

Problem 2.3 *Calculate the pressure due to a column of 0.3 of (a) water, (b) an oil of sp. gr. 0.8, and (c) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000$ kg/m^3.*

Solution. Given :

Height of liquid column, $Z = 0.3$ m.

The pressure at any point in a liquid is given by equation (2.5) as

$$p = \rho g Z$$

(*a*) For water, $\rho = 1000$ kg/m^3

$$\therefore \quad p = \rho g Z = 1000 \times 9.81 \times 0.3 = 2943 \text{ N/m}^2$$

$$= \frac{2943}{10^4} \text{ N/cm}^2 = \mathbf{0.2943 \text{ N/cm}^2}. \textbf{ Ans.}$$

(*b*) For oil of sp. gr. 0.8,

From equation (1.1*A*), we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water.

$\therefore$ Density of oil, ρ_0 = Sp. gr. of oil × Density of water (ρ_0 = Density of oil)

$$= 0.8 \times \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

Now pressure, $p = \rho_0 \times g \times Z$

$$= 800 \times 9.81 \times 0.3 = 2354.4 \frac{\text{N}}{\text{m}^2} = \frac{2354.4}{10^4} \frac{\text{N}}{\text{cm}^2}.$$

$$= \mathbf{0.2354 \frac{N}{cm^2}}. \textbf{ Ans.}$$

(*c*) For mercury, sp. gr. = 13.6

From equation (1.1*A*) we know that the density of a fluid is equal to specific gravity of fluid multiplied by density of water

$\therefore$ Density of mercury, ρ_s = Specific gravity of mercury × Density of water

$$= 13.6 \times 1000 = 13600 \text{ kg/m}^3$$

$$\therefore \quad p = \rho_s \times g \times Z$$

$$= 13600 \times 9.81 \times 0.3 = 40025 \frac{\text{N}}{\text{m}^2}$$

$$= \frac{40025}{10^4} = \mathbf{4.002 \frac{N}{cm^2}}. \textbf{ Ans.}$$

Problem 2.4 *The pressure intensity at a point in a fluid is given 3.924 N/cm². Find the corresponding height of fluid when the fluid is : (a) water, and (b) oil of sp. gr. 0.9.*

Solution. Given :

Pressure intensity, $p = 3.924 \dfrac{\text{N}}{\text{cm}^2} = 3.924 \times 10^4 \dfrac{\text{N}}{\text{m}^2}$.

The corresponding height, Z, of the fluid is given by equation (2.6) as

$$Z = \frac{p}{\rho \times g}$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$$\therefore \quad Z = \frac{p}{\rho \times g} = \frac{3.924 \times 10^4}{1000 \times 9.81} = \textbf{4 m of water. Ans.}$$

(*b*) For oil, sp. gr. $= 0.9$

$\therefore$ Density of oil $\rho_0 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$$\therefore \quad Z = \frac{p}{\rho_0 \times g} = \frac{3.924 \times 10^4}{900 \times 9.81} = \textbf{4.44 m of oil. Ans.}$$

Problem 2.22 (SI Units) *If the atmosphere pressure at sea level is 10.143 N/cm², determine the pressure at a height of 2500 m assuming the pressure variation follows (i) Hydrostatic law, and (ii) isothermal law. The density of air is given as 1.208 kg/m³.*

Solution. Given :

Pressure at sea-level, $p_0 = 10.143 \text{ N/cm}^2 = 10.143 \times 10^4 \text{ N/m}^2$

Height, $Z = 2500$ m

Density of air, $\rho_0 = 1.208 \text{ kg/m}^3$

(*i*) **Pressure by hydrostatic law.** For hydrostatic law, ρ is assumed constant and hence p is given by equation $\frac{dp}{dZ} = -\rho g$

Integrating, we get $\int_{p_0}^{p} dp = \int -\rho g \, dZ = -\rho g \int_{Z_0}^{Z} dZ$

or $p - p_0 = -\rho g [Z - Z_0]$

For datum line at sea-level, $Z_0 = 0$

∴ $p - p_0 = -\rho g Z$ or $p = p_0 - \rho g Z$

$$= 10.143 \times 10^4 - 1.208 \times 9.81 \times 2500 \quad [\because \rho = \rho_0 = 1.208]$$

$$= 101430 - 29626 = 71804 \frac{\text{N}}{\text{m}^2} \text{ or } \frac{71804}{10^4} \text{ N/cm}^2$$

$$= \mathbf{7.18 \text{ N/cm}^2. \text{ Ans.}}$$

(*ii*) **Pressure by Isothermal Law.** Pressure at any height Z by isothermal law is given by equation (2.18) as

$$p = p_0 \, e^{-gZ/RT}$$

$$= 10.143 \times 10^4 \, e^{-\frac{gZ \times \rho_0}{p_0}} \qquad \left[\because \frac{p_0}{\rho_0} = RT \text{ and } \rho_0 g = w_0\right]$$

$$= 10.143 \times 10^4 \, e^{-\frac{Z\rho_0 \times g}{p_0}}$$

$$= 10.143 \times 10^4 \, e^{(-2500 \times 1.208 \times 9.81)/10.143 \times 10^4}$$

$$= 101430 \times e^{-.292} = 101430 \times \frac{1}{1.3391} = 75743 \text{ N/m}^2$$

$$= \frac{75743}{10^4} \text{ N/cm}^2 = \mathbf{7.574 \text{ N/cm}^2. \text{ Ans.}}$$

Problem 2.24 *Calculate the pressure at a height of 7500 m above sea level if the atmospheric pressure is 10.143 N/cm² and temperature is 15°C at the sea-level, assuming (i) air is incompressible, (ii) pressure variation follows isothermal law, and (iii) pressure variation follows adiabatic law. Take the density of air at the sea-level as equal to 1.285 kg/m³. Neglect variation of g with altitude.*

Solution. Given :

Height above sea-level, $Z = 7500$ m

Pressure at sea-level, $p_0 = 10.143 \text{ N/cm}^2 = 10.143 \times 10^4 \text{ N/m}^2$

Temperature at sea-level, $t_0 = 15°\text{C}$

∴ $T_0 = 273 + 15 = 288°\text{K}$

Density of air, $\rho = \rho_0 = 1.285 \text{ kg/m}^3$

(*i*) *Pressure when air is incompressible :*

$$\frac{dp}{dZ} = -\rho g$$

∴ $$\int_{p_0}^{p} dp = -\int_{Z_0}^{Z} \rho g dz \quad \text{or} \quad p - p_0 = -\rho g[Z - Z_0]$$

or $$p = p_0 - \rho g Z \qquad \{\because \quad Z_0 = \text{datum line} = 0\}$$

$$= 10.143 \times 10^4 - 1.285 \times 9.81 \times 7500$$

$$= 101430 - 94543 = 6887 \text{ N/m}^2 = \mathbf{0.688 \ \frac{N}{cm^2}}. \textbf{ Ans.}$$

(*ii*) *Pressure variation follows isothermal law :*

Using equation (2.18), we have $p = p_0 e^{-gZ/RT}$

$$= p_0\, e^{-gZ\rho_0/p_0} \qquad \left\{\because\ \frac{p_0}{\rho_0} = RT \ \therefore\ \frac{\rho_0}{p_0} = \frac{1}{RT}\right\}$$

$$= 101430\ e^{-gZ\rho_0/p_0} = 101430\ e^{-7500\,\times\,1.285\,\times\,9.81/101430}$$

$$= 101430\ e^{-.9320} = 101430 \times .39376$$

$$= \mathbf{39939\ N/m^2\ or\ 3.993\ N/cm^2.\ Ans.}$$

(*iii*) *Pressure variation follows adiabatic law :* [k = 1.4]

Using equation (2.19), we have $p = p_0\left[1 - \frac{k-1}{k}\, gZ\, \frac{\rho_0}{p_0}\right]^{k/(k-1)}$, where ρ_0 = 1.285 kg/m^3

$$p = 101430\left[1 - \frac{(1.4-1.0)}{1.4} \times 9.81 \times \frac{(7500 \times 1.285)}{101430}\right]^{\frac{1.4}{1.4-1.0}}$$

$$= 101430\ [1 - .2662]^{1.4/.4} = 101430 \times (.7337)^{3.5}$$

$$= \mathbf{34310\ N/m^2\ or\ 3.431\frac{N}{cm^2}.\ Ans.}$$

Problem 4.8 *A uniform body of size 3 m long × 2 m wide × 1 m deep floats in water. What is the weight of the body if depth of immersion is 0.8 m ? Determine the meta-centric height also.*

Solution. Given :

Dimension of body $= 3 \times 2 \times 1$

Depth of immersion $= 0.8$ m

Find (*i*) Weight of body, *W*

(*ii*) Meta-centric height, *GM*

(*i*) **Weight of Body, W**

$= $ Weight of water displaced

$= \rho g \times$ Volume of water displaced

$= 1000 \times 9.81 \times$ Volume of body in water

$= 1000 \times 9.81 \times 3 \times 2 \times 0.8$ N

$=$ **47088 N. Ans.**

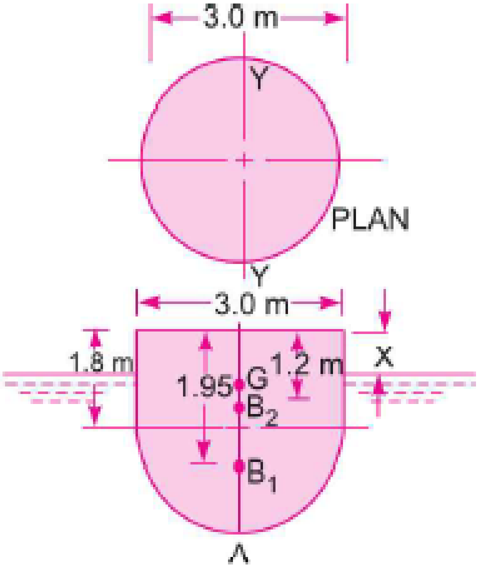

Fig. 4.8

(*ii*) **Meta-centric Height, GM**

Using equation (4.4), we get

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I about *Y-Y* axis of the plan of the body

$$= \frac{1}{12} \times 3 \times 2^3 = \frac{3 \times 2^3}{12} = 2.0 \text{ m}^4$$

$\forall$ = Volume of body in water

$= 3 \times 2 \times 0.8 = 4.8 \text{ m}^3$

$$BG = AG - AB = \frac{1.0}{2} - \frac{0.8}{2} = 0.5 - 0.4 = 0.1$$

$$\therefore \quad GM = \frac{2.0}{4.8} - 0.1 = 0.4167 - 0.1 = \mathbf{0.3167 \text{ m. Ans.}}$$

Thank You